Experimental Verification of Bayesian Planet Detection Algorithms with a Shaped Pupil Coronagraph
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Problem

Simulation of star and planet images with a shaped pupil coronagraph [Kasdin et al., 2003]. Planet is 10^3 times dimmer than star, and ~ 4 times brighter than speckle average, but is still difficult to pick out from speckles. Image plane figures are Log(Intensity).

Linear Filters and Cross-Correlation

- Model observation as:
  \[ z(x, y) = C_p(x, \xi, \eta) \cdot \mathbf{P} + \nu \]
  where \( C_p \) is the mean photon count at planet location - pixel \((\xi, \eta)\), \( \mathbf{P} \) is the normalized PSF, and \( \nu \) is the noise. [Kasdin and Braems, 2006]
- Seek filter \( h \) to maximize signal-to-noise (SNR): \( \text{SNR} = \frac{\mathbb{E} [\langle h, C_p \rangle^2]}{\mathbb{E} [h^2]} \)
  where \( \langle \cdot, \cdot \rangle \) is the inner product, \( C \) is the expectation and \( s = \langle h, C_p \rangle \) and \( n = \mathbb{E} [h^2] \)
- The optimal (matched) filter is then \( h = \mathbb{R}^{-1} C_p \cdot \mathbf{P} \) for constant \( s \) and noise covariance (or autocorrelation) \( R \), with filter output given by the convolution: \( y = h * z \)
- A matched filter can be replaced by correlation operations for template matching. [Ziemer and Tranter, 2002]
- Define a normalized cross-correlation as:
  \[ \gamma = \frac{\langle z - T \rangle \cdot \langle T - \bar{T} \rangle}{\sqrt{\langle (z - T) \cdot (z - T) \rangle \cdot \langle (T - \bar{T}) \cdot (T - \bar{T}) \rangle}} \]
  where \( T \) is the template, \( z' \) is the section of the image beneath the template, and \( \bar{x} \) denotes normalization. [Lewis, 1995]

Simulation

Simulation of cross-correlation applied to planet image with no speckle (and star removed with focal plane mask) using normalized PSF as the template. Template and image are Log(Intensity). Corner peaks are due to edge effects.

Simulation of cross-correlation applied to planet image with speckle. Planet is at 99% mean speckle intensity and second cross-correlation peak is due to strong speckle.

How do we determine significance of a peak?

Express the filtered observation as:
\[ y = (k + f) \cdot h + \nu + h \]
where \( k \) is the optical system impulse response, \( f \) is the original pattern and \( P_h = k \cdot h \).
[Navarro et al., 2004]
- Assuming a constant prior for impulse response, the posterior for the input pattern is:
  \[ p(f \mid C_p(\xi, \eta)) = \mathbb{R}(f \cdot C_p(\xi, \eta)) \cdot p(f) \]
- Seek the probability that the input pattern matches the template:
  \[ p(f \mid T \cdot C_p(\xi, \eta)) \]
- Maximizing this probability is equivalent to minimizing Euclidean distance error function so:
  \[ \max p(f \mid T \cdot C_p(\xi, \eta)) \exp \left( \mathbb{R}^{-1} \cdot \langle \gamma \rangle \cdot \gamma \right) \]
- Choose threshold for cross-correlation based on desired minimum probability of match.

References

- [Groff et al., 2010]

Lab Setup

Separate laser sources are used to generate the star and planet, one transmitted and one reflected through a 0.5° glass wedge to a collimating optic (OAP 1 in the figure below).

High Contrast Imaging Lab (HCIL), [Groff et al., 2010]

Experimental Results

Top row: lab images with and without planet signal. Bottom row: filter applied to areas in black boxes with stated probability threshold. Images are linearly scaled.

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